Mean-field models for fishery management

Mean Field Games, introduced in the seminal works [5, 1], aim at studying statistical limit systems of deterministic or stochastic differential games as the number of agents tends to infinity.

This project focuses on a model for fishery management, introduced in [3, 4], centered on systems of coupled partial differential equations. As is customary in the applied mathematical literature of spatial ecology, the fish dynamics are modelled through a reaction-diffusion equation, with an additional term representing the fishing effort of the mass of fishermen. The dynamics of the fishermen are derived using the theory of Mean Field Games: we assume that each fisherman tries to maximise their own individual profit, and that they are numerous enough so that the modelling error due to the mean-field limit is small.

The regime that has been mainly studied up to now is the competitive one, which may lead to extinction or depletion of the population of fishes, see [3], and in turn to low profit for the fishermen. Such collective outcomes are usually dubbed *loose-loose* in the framework of games. In [2], we make a numerical comparison between such a competitive equilibrium and the solution of a cooperative games (where the fishermen aim at maximising the total catch) and we observe an increase, up to a multiplicative factor of about four, of the total number of living fishes in the latter case, as well as an improvement of the individual profits of all fishermen.

In the present project, we assume that an organisation ruling the fishery may change the tax scheme in order to incentivise a mass of competitive fishermen to change their behaviour to reduce their negative effect on the fish population. By adding a layer of optimisation over the choice of the tax, we can derive a new system of coupled partial differential equations that fits within the framework of Stackelberg Mean Field Games. The objectives of this project are as follows:

- rigorously justify the derivation of the PDE system,
- study the existence of solutions to such a system (uniqueness may be considered in some particular cases, but we believe it to be much more involved in general),
- develop numerical schemes to compute approximate solutions and compare them with those of the competitive system.

References

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