1 Numerical Methods for Stackelberg Mean Field Games with Common Noise

Mean field games were introduced in the mid-2000s by Lasry and Lions [6, 7], and independently by Huang, Malhamé, and Caines [5], to provide tractable approximations of Nash equilibria in large-population games. The Stackelberg structure refers to the presence of a leader who anticipates and influences the behavior of the large population in order to optimize a criterion that depends on the resulting mean field game equilibrium. Such models, commonly referred to as Stackelberg mean field games [3], are well suited for situations in which a regulator or planner seeks to steer the decisions of a large group of strategic agents. Typical applications include grid regulations in electricity markets [1] and carbon emission taxing [2] and regulation of certificate markets [4]. In many relevant settings, realism requires the inclusion of aggregate shocks (such as the weather effects), modeled through a common source of randomness affecting the entire population. However, computing solutions to Stackelberg mean field games with common noise remains a challenging problem, and there are currently no widely accepted generic and efficient numerical methods. The present project aims to develop such methods, with a particular focus on applications related to the environmental transition.

More specifically, we will consider a model in which a green regulator influences a large population of energy producers. The producers solve a mean field game whose cost function is affected by the regulator's decisions, which includes, for instance, tax levels or incentives to invest in renewable energy sources. In this setting, common noise arises from natural phenomena impacting all energy producers, such as weather variations. Owing to the high complexity and dimensionality of the resulting model, classical numerical methods are not applicable. The project therefore explores machine learning approaches based on neural networks, motivated by their potential scalability.

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