Numerical Methods for Credit Risk Management under Stochastic Climate Scenarios and Model Uncertainty

Scientific context and objectives

Climate risk assessment for large credit portfolios lies at the intersection of supervisory climate stress testing, internal risk management, and model risk governance. Supervisory exercises rely on scenario-based climate narratives and portfolio-level aggregation of losses [8, 2]. Supervisory guidance explicitly emphasizes governance and robustness of climate-related risk measurement [3].

A standard modelling chain starts from NGFS/SSP scenario trajectories and maps physical and transition drivers to corporate credit risk [16]. In particular, linking socioeconomic transition pathways (SSP-like narratives) to firms' dynamics and credit metrics has been formalized in portfolio credit settings [5, 4, 15]. Beyond the "mean path", practical risk analysis requires a panel of stochastic scenarios (variability, correlations, extremes) rather than a single deterministic trajectory.

A second layer of complexity arises from production defaults: temporary or persistent breakdowns in productive capacity induced by extreme physical events, energy supply disruptions, or binding transition constraints. Such events affect cash flows upstream of credit defaults and introduce regime switching, discontinuities and path dependence.

Finally, transition risk is subject to deep (Knightian) uncertainty, especially regarding future carbon pricing. Ambiguity-aware corporate credit models have been proposed to capture this dimension [7, 19].

From a mathematical viewpoint, the resulting framework combines high-dimensional stochastic control and switching dynamics with robustness requirements [10, 18, 17, 20]. The objective of this project is not to introduce new economic mechanisms, but to use existing climate-to-credit model structures while developing numerically tractable, scalable, and auditable methods for (i) stochastic scenario generation, (ii) large-scale simulation, and (iii) robustness quantification of tail risk measures.

Modelling framework (fixed)

A portfolio of obligors is exposed to climate risk through three interacting channels:

- **Physical risk**: SSP-consistent climate trajectories mapped to output/cash-flow via IAM-inspired (DICE-type) damage functions;
- **Transition risk**: firms optimally adjust production levels and energy mix under emissions constraints consistent with SSP objectives, under demand uncertainty;

- **Production defaults**: temporary or permanent production failures triggered by extreme shocks, energy shortages, or binding transition constraints.

Production defaults induce regime switching and non-smooth cash-flow dynamics, naturally leading to controlled Markov processes with switching regimes (and potentially jumps) [17, 18]. All modelling assumptions are fixed; the project focuses on numerical methods for the induced high-dimensional, non-smooth stochastic system.

Numerical challenges and CEMRACS contributions

1) Stochastic scenario generation consistent with NGFS/SSP narratives

NGFS/SSP pathways are largely deterministic. For portfolio loss analysis, they must be embedded into **stochastic processes** preserving temporal coherence, cross-risk correlations, and tail behaviour relevant for extremes and production defaults. Reduced representations (Karhunen–Loève, dynamic PCA, polynomial chaos / surrogates) will be explored, guided by NGFS documentation [16].

2) Transition risk and production defaults as stochastic control problems

Each obligor faces a long-horizon stochastic control problem under constraints. Production defaults introduce regime switching and non-smooth value functions, placing the problem within the framework of viscosity solutions for HJB-type equations [10, 18]. Numerically, the project investigates regression-based and deep-learning-based policy approximations for high-dimensional stochastic control, with theoretical convergence analysis as in [14].

Recent advances in machine learning for stochastic control provide scalable alternatives to classical grid-based numerical schemes in high-dimensional settings. Deep BSDE methods allow to approximate value functions of nonlinear HJB equations arising from long-horizon climate transition problems, with provable convergence properties [12, 1, 13]. Actor–critic and reinforcement learning approaches provide direct approximations of optimal policies, including in the presence of jumps and regime-switching dynamics induced by production defaults [21, 11, 6]. These methods naturally accommodate large state spaces and scenario-dependent controls, making them well suited for portfolio-scale climate stress testing.

3) Scalable simulation of regime-switching production processes

Stochastic climate factors induce correlated regime switches and nonlinear production cash flows across large credit portfolios. When combined with long horizons and scenario ensembles, naive Monte Carlo simulation becomes computationally prohibitive.

This work package focuses on scalable numerical simulation strategies, combining classical Monte Carlo methodology with controlled machine-learning-based accelerations:

- variance reduction techniques (control variates, importance sampling) enhanced by data-driven learning of optimal controls while preserving unbiasedness;
- amortized estimation of regime-switching mechanisms conditional on climate states, using constrained regression or classification techniques to reduce repeated simulation costs;
- diagnostics for numerical stability and variance explosion induced by rare regime-switching events.

The objective is to enable large-scale simulation of regime-switching production processes without altering the underlying probabilistic structure of the model.

4) High-dimensional dependence and scenario-conditional factor reduction

Climate drivers, production defaults, and credit defaults jointly shape portfolio dependence in a high-dimensional and nonlinear manner. Direct modelling quickly becomes unstable, while naive factor reduction may severely distort tail dependence and risk measures.

The project investigates scenario-conditional factor reduction methods supported by numerical and AI-based tools:

- dynamic and scenario-dependent factor representations adapted to stochastic climate trajectories;
- constrained linear and weakly nonlinear dimension reduction techniques (e.g. regularized auto-encoding) designed to preserve monotonicity and tail behaviour;
- systematic analysis of the stability of VaR and Expected Shortfall under factor truncation;
- comparison with classical copula-based and factor-copula approaches.

A key objective is to reduce effective dimensionality while maintaining robustness of extreme risk measures.

5) Fast approximation of portfolio loss distributions

Repeated brute-force Monte Carlo simulations are infeasible for large portfolios and extensive sensitivity analysis. Fast and controlled approximations of portfolio loss distributions are therefore required.

This work package develops hybrid surrogate approaches combining numerical analysis and machine learning:

- sparse polynomial chaos expansions adapted to stochastic climate scenarios, with data-driven selection of dominant terms [9];
- hybrid surrogates where learning-based models approximate only the residual structure not captured by polynomial representations;
- tail-focused training strategies prioritizing accuracy for high quantiles and Expected Shortfall rather than mean-square error;
- validation protocols explicitly targeting stability of tail risk measures.

The goal is to obtain fast, interpretable approximations of loss distributions suitable for stress testing and robustness analysis.

6) Robustness, numerical uncertainty, and model risk

This work package produces audit-ready robustness outputs: sensitivity of VaR/Expected Shortfall to (i) scenario stochasticisation choices, (ii) regime specification / thresholds for production defaults, (iii) factor truncation, and (iv) surrogate accuracy. Robustness with respect to ambiguous transition parameters (e.g. carbon pricing) will be addressed using Knightian uncertainty approaches in corporate credit risk [7, 20]. The goal is to separate economic climate risk from numerical artefacts via uncertainty decomposition and error budgets.

Expected outcomes

- Scalable numerical pipeline for climate stress testing under a panel of stochastic scenarios;
- Approximation schemes for high-dimensional stochastic control with regime switching;
- Robustness diagnostics compatible with supervisory and internal model risk governance;
- Reproducible numerical benchmarks (portfolio size, scenarios, runtime/accuracy trade-offs).

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